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6 An introduction to logical nihilism

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Abstract. The spectre of logical nihilism—the view that there is no logic—has arisen recently in the literature on logical pluralism. Pluralists say that there is more than one correct logic, monists that there is only one. But could there be none at all? This paper presents an argument for the view that there is no logic, offering counterexamples even to apparently secure principles like conjunction elimination and identity. It develops a nihilist first order model theory, and which then reveals one problem with the argument for logical nihilism—though not, it is argued, one which should comfort the anti-nihilist

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Logical nihilism is the view that there is no logic. It's the limit on a spectrum which contains logical monism—the traditional view that there is exactly one logic—and logical pluralism—the view that there is more than one, popularised by (Beall & Restall, 2006) as well as (Varzi, 2002), (Field, 2009), and (Shapiro, 2014).[1] There are

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[1]As pluralists usually note, there are many ways to understand "logic" and on some senses it is quite trivial that there are many logics. The dispute gets interesting when we reserve "logic" for specifications of the entailment relation on a language and count specifications which agree on the extension of that relation as agreeing on the logic. The pluralist is then committed to the existence of many entailment relations, and (my kind of) nihilist to that relation having no relata. There is not much recent mainstream literature on nihilism, but Aaron Cotnoir has kindly showed me a draft of his unpublished paper on this topic. Mortensen, 1989 argues for two relatives of logical nihilism, one based on the idea that nothing is necessary, the other on the idea that nothing is true in all mathematical models. Mortensen, on my reading, is a nihilist about logical truth, though I am unsure whether he would generalise this to logical consequence. Nihilism is

a few ways to make the idea more precise, but on my favoured interpretation, logical nihilism says that there are no *laws* of logic, i.e. no pairs of premise-sets and conclusions such that the premises logically entail the conclusion, or equivalently, that the extension of the *logical entailment* relation is the empty set. The nihilist says that no mater what Γ and φ are,

$$\Gamma \nvDash \varphi.$$

This might seem extreme. Many non-classical logicians reject particular laws of logic, such as the law of excluded middle or explosion, but they are usually at pains to hang on to the laws that remain; overly weak logics threaten to be neither interesting nor useful. The nihilist gives up on *all* of these laws, including modus ponens and the principle of non-contradiction ($\vDash \neg(\varphi \wedge \neg\varphi)$) as well as less controversial principles, such as conjunction elimination ($\varphi \wedge \psi \vDash \varphi$) and identity ($\varphi \vDash \varphi$).

In this short paper I am going to argue that the view is much more reasonable than it might at first seem, present one argument for nihilism—including counterexamples to supposedly "safe" laws like conjunction elimination and identity—and show how to give a nihilist first-order model theory. This will reveal one limitation of the argument for nihilism, but it is not one, I will argue, which should comfort the anti-nihilist.

1 Nihilism, generality and self-defeat

Philosophers sometimes object to nihilism on the grounds that it is that it is self-defeating. The idea is that any argument for nihilism must *use* logic, and so if the conclusion is true, the argument for it must have used something to which it was not entitled; the truth of the conclusion would undermine the argument's force.

In response I wish to point out that even if there are no logical *laws*, it can still be the case that particular instances of familiar forms are unproblematic. For example, this instance of modus ponens

Snow is white $\rightarrow$ grass is green.
Snow is white.
____
Grass is green

might be perfectly acceptable to the nihilist—in the sense that the truth of the premises guarantees that of the conclusion—even if she believes that modus ponens is not a law of logic, thanks to some recherché counterexamples involving the truth predicate.

also touched on in (Bueno & Shalkowski, 2009) and discussed in terms of models in a recent paper by Estrada-González, 2012.

To claim that modus ponens is *logically* valid is to make a claim of great generality, whereas to claim that the argument above preserves truth is merely to claim that some *instance* of modus ponens is good. The nihilist can accept many instances of modus ponens, so long as they also think that there are some instances which are mistaken.

An analogy with intuitionism or dialetheism can be useful here: intuitionists deny that the law of excluded middle is a law of logic, but even they can reasonably appeal to it when not concerned with statements about infinite collections. (Iemhoff, R., 2013) And dialetheists reject explosion ($\varphi, \neg\varphi \vDash \psi$) as a law of logic—they might cite counterexamples that substitute Liar sentences for φ—and yet they may still think that it is ok to use explosion when we are doing simple number theory, or working out what to have for dinner. In a similar spirit, a nihilist can allow that all the moves in an argument are just fine—they never take one from truth to falsehood—even while they deny that they are instances of completely general, counterexample-free laws of logic. In fact, the generality of logic can make nihilism more appealing. Who is to say that further research might not give us good reason to think that there are sentences which, when substituted for φ and ψ in some erstwhile law, give us true premises but a non-true conclusion?[2]

2 An argument for logical nihilism

It is one thing to show that a view is not self-defeating, and another to give a positive argument for it. Our argument *for* nihilism begins by noting that if we artificially limit the substitution class of sentences, the result can be an artificially strong logic—one which weakens as soon as we lift the restrictions. For example, suppose we were only allowed to substitute *true* sentences for φ and ψ in the fallacious argument form $\varphi \to \psi, \psi \vDash \varphi$ (affirming the consequent). Whatever we substituted for the conclusion φ would be true and the fallacious principle would appear valid.

To get a more accurate picture we need to lift any artificial restrictions on what we substitute for sentence variables. In this case, we need to be able to substitute sentences which have the truth-value *false*, and this will allow us to substitute a true sentence for ψ and a false one for φ, obtaining a counterexample.

But have we have lifted the restrictions far enough? Consideration of sentences containing empty names, or predicates with indeterminate extension, or even future con-

[2] One might be tempted to say that it is the fact that we have *proved*, say, the law of excluded middle that rules out the possibility of later counterexamples, no matter how general the theorem is. But the usual model theoretic proofs of, say, the law of excluded middle are not held to be proofs by intuitionists; (moreover a certain kind of intuitionist thinks there are Kripke-style models which are counterexamples to the law of excluded middle—these are simply not considered to be counterexamples by the classical logicians.) Anyway, proofs of logical laws do not generally have the status required to settle the matter.

tingents, leads some philosophers to think that some sentences are *neither* true nor false, and so perhaps we should allow substitution of sentences which are *neither*. We might adjust our logic by adopting the strong Kleene valuations for the connectives and now it seems clear that if we substitute a *neither* sentence for φ in $\varphi \vee \neg\varphi$ we get an instance of the law of excluded middle which is not true—and hence that principle is not a logical truth after all.[3] Indeed, this approach has the same consequences for all laws in which there is nothing to the left of the turnstile including $\vDash \varphi \to \varphi$ and $\vDash \neg(\varphi \wedge \neg\varphi)$.

We still have the laws with something on the left, including modus ponens and disjunctive syllogism. But the dialetheist thinks that we only have some of these because we are still artificially restricting our substitution class. She thinks that once the language gets rich enough—say, admitting of truth and other semantic predicates—it becomes possible for sentences to be both true and false. (Priest, 1987/2006) Allowing such substitutions gives us counterexamples to both disjunctive syllogism and modus ponens.[4]

Combining the Strong Kleene and LP valuations of the connectives, the result of allowing sentences to be *both* and *neither* is the logic of First Degree Entailment (FDE). FDE is closer to logical nihilism than either LP or K_3, but it is not nihilism yet. Among the laws FDE retains are:

$$\begin{array}{rcll}
\varphi \wedge \psi & \vDash & \psi & (\wedge E) \\
\varphi, \psi & \vDash & \varphi \wedge \psi & (\wedge I) \\
\varphi & \vDash & \varphi \vee \psi & (\vee I) \\
\varphi & \vDash & \varphi & (ID)
\end{array}$$

Can the substitution class can be widened yet further to allow counterexamples to even these? I plan to show that it can and I will focus my attention on $(\wedge E)$ and (ID), on the grounds that these are sometimes take to be the safest and most secure laws of logic. The thought will be that if we can given counterexamples to even these principles, then we can give counterexamples to any logical laws.[5]

[3] I assume throughout this paper that the designated values are the ones that include truth e.g. *true* and *both*.

[4] For both (DS) and (MP) we can let φ be *both* and ψ be *false*. Then all the premises come out true (though some may be false as well) and the conclusions are false.

[5] Conjunction elimination is so sacrosanct—or perhaps just so uninteresting—that even the most radical of logicians sometimes see it as secure: “I think it just false that all principles of inference fail in some situation. For example, any situation in which a conjunction holds, the conjuncts hold, simply in virtue of the meaning of $\wedge$." Priest, 2006, p. 202–3 Perhaps that it because it is analytic, or true in virtue of the meaning of ‘and’. But claims of analyticity in the formal sciences have a bad track record. We’re often less confident of our claims about meaning (and so claims about truth in virtue of meaning) claims than we are of the basic principles of logic. “Attempts are made to fence off purely logical claims as in some sense *analytic*, in a sense that would make them uncontroversial, whereas metaphysical claims are correspondingly synthetic, and inherently liable to controversy. The history of logic tells against any such

($\wedge E$) is sometimes taken to be definitional of $\wedge$. (Carnap, R., 1937, p. xv) (Gentzen, G., 1964) And even if it isn't, one might think that the reoccurrence of φ on the left and the right of the turnstile, along with the standard truth-conditions for conjunction, are sufficient for the correctness of ($\wedge E$). If $\varphi \wedge \psi$ is true, then the truth-conditions for $\wedge$ say that φ is true, giving us the truth of the right-hand side.

There are two ways a nihilist might try to resist here. The first is to dispute the definition and/or truth-clause for $\wedge$. There are logics where conjunction is given an alternative (non-equivalent) definition, and one might even think that one of these gives a better account of the truth-conditions of the English word 'and'. (Restall, 2002) Two problems with with this response are, first, that it is vulnerable to the charge of 'changing the subject'; the question was about ($\wedge E$), not a different rule written with similar looking symbols. (Quine, 1986) The second problem is that that the new logic will have laws for $\wedge$ too—just different ones—and so this strategy would remove obstacles to nihilism only to set up new ones.

A more promising approach is to allow the substitution of some odd kinds of expression, for example, to get a counterexample to conjunction elimination we might consider sentences whose truth-value depended upon *whether or not they were in the scope of a conjunction.* Imagine, for instance, that we had a predicate 'con-white' which took the same extension as 'white' while embedded in a conjunction, but the null-set when not so embedded. Then the following argument would have true premises and a false conclusion:

$$\frac{\text{Snow is con-white} \wedge \text{grass is green.}}{\text{Snow is con-white.}}$$

Using a similar strategy to get a counterexample to (ID), we might consider sentences whose truth-value depends upon whether they function as a premise or as a conclusion in an argument. Suppose we had a predicate *prem-white*, whose extension matches the extension of *white* when the sentence appears in the premises to an argument, but is the null set otherwise. Then the following argument would have a true premise but a false conclusion:

$$\frac{\text{Snow is prem-white.}}{\text{Snow is prem-white.}}$$

We are quite used to considering context and how it effects the truth-value of sentences and their logic, though usually the aspects of context we are interested in are agent-identity and the time or place of the speech act—things which do not change across

contrast. All major logical principles have been rejected on metaphysical grounds."Williamson, 2013, p. 146

the argument. (Kaplan, 1989) The proposal now is simply to look at expressions whose extension varies with *sentential context*, which may.[6] In the next section we will look at a way to introduce predicates like this into the model theory for a standard first-order logic.

3 Nihilist model theory

We will introduce the new models by making adjustments to the interpretation function of a classical, first-order Tarski model. We begin with a simple first-order language, whose non-logical expressions are individual constants (a, b, c, ...etc.) and predicates of arity $n > 0$ (P, Q, R,...etc). The logical expressions are the truth-functors $\neg, \wedge, \vee$ and $\rightarrow$, and the quantifier $\forall$. We will also need individual variables, x, y, z, etc. Models are pairs of a domain and an interpretation function, $\langle D, I\rangle$, with D a non-empty set and I a function which assigns extensions to all the non-logical expressions (elements of the domain to individual constants and elements of D^n to non-logical predicates of arity $n > 0$.)

In classical models the truth-values of atoms are determined independently of their position in any larger sentence. That is the natural result of the fact that interpretation functions in classical models do not take sentence position, or anything that varies with it, as an argument. We can change that. But first, some terminology: let a predicate's *atom* be the shortest formula of which it is a part. An atom is in the *direct scope* of a truth-functor iff the smallest *proper* super-formula containing the atom has that functor as its main connective.[7] For example, in the formula $Pa \wedge Qb$, P's atom is Pa, which is in the direct scope of the conjunction. In the formula Pa, P's atom is Pa and it is not in the direct scope of any truth-functor, and in $\neg(Pa \rightarrow (Qb \wedge Rc))$, P's atom is Pa and this is in the direct scope of the conditional, but not in the direct scope of $\neg$ or $\wedge$.

For our first counterexample we need only three sentence position values: L, R, and S. L is the position of a predicate whose atom is in the direct scope of some binary truth-functor and whose atom appears to the left of that truth-functor. R is the position of a predicate whose atom is in the direct scope of some binary truth-functor and whose atom appears to the right of that truth-functor. And S is for a predicate whose atom is not in the direct scope of any binary truth-functor.[8] Now let $I(\alpha, X)$ (the interpretation

[6] The idea of such sensitivity to sentential context can already be found in (Frege, 1985/1892); Frege thinks that the extension of a sentence varies depending on whether or not it is embedded in a propositional attitude ascription.

[7] The smallest superformula containing the atom is the atom itself, but that is not a proper superformula.

[8] For the purposes of these definitions we assume that our truth-functors are $\leq$ 2-place, and that binary functors are written between their arguments. We would probably want numerical position-values if we were dealing with languages which contained $>$ 2-place truth-functors.

function of the model) be a function which takes a non-logical expression, α, and a sentence position, X, and yields an appropriate extension.

Now I could be a constant function of sentence position for some predicates. For example, where the domain is the natural numbers, this is a possible interpretation for a predicate P:

$$I(P,L) = \{0,1,2\}$$
$$I(P,R) = \{0,1,2\}$$
$$I(P,S) = \{0,1,2\}$$

Here the position of P's sentence atom makes no difference to P's extension. But we can also have new kinds of predicates, such as *LeftP*, *RightP*, and *SoloP*, whose extension varies with sentence position:

$I(LeftP,L) = \{0,1,2\}$	$I(RightP,L) = \emptyset$	$I(SoloP,L) = \emptyset$
$I(LeftP,R) = \emptyset$	$I(RightP,R) = \{0,1,2\}$	$I(SoloP,R) = \emptyset$
$I(LeftP,S) = \emptyset$	$I(RightP,S) = \emptyset$	$I(SoloP,S) = \{0,1,2\}$

We extend I to a valuation function V with the following definition:

1. $V(\Pi t_1,...t_n) = 1$ iff $I(\langle[t_1],....[t_n]\rangle) \in I(\Pi,X)$, where Π is an n-place predicate and $t_1,...,t_n$ are terms.[9]
2. $V(\neg\varphi) = 1$ iff $V(\varphi) = 0$
3. $V(\varphi \wedge \psi) = 1$ iff $V(\varphi) = 1$ and $V(\psi) = 1$
4. $V(\varphi \rightarrow \psi) = 1$ iff $V(\varphi) = 0$ or $V(\psi) = 1$
5. $V(\varphi \vee \psi) = 1$ iff $V(\varphi) = 1$ or $V(\psi) = 1$ or both
6. $V(\forall\xi\varphi) = 1$ iff for all $u \in D$, $V_{g^u_\xi}(\varphi) = 1$[10]

With the exception of clause 1. which mentions the new style of interpretation function, these are the familiar classical truth clauses; no mention is made of the sentence position variable. But since the extension of a predicate, and hence the truth-value of an atomic sentence, can depend on sentence position, the truth-value of a complex sentence can still be affected by sentence position. It's conceivable, for example, that $Fa \wedge Gb$ could be true, while $Gb \wedge Fa$ is false.[11]

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[9] Atomic formulas do not officially have parentheses, but I will sometimes add them for readability.

[10] g is a variable assignment and g^u_ξ one just like g except that the element $u \in D$ is assigned to the variable ξ. I will say more about variable assignments in the next section, but the details can be ignored until then.

[11] This would give us a counterexample to the commutativity of conjunction: $Left(a) \wedge Right(b) \nvDash Right(b) \wedge Left(a)$. It would model the invalidity of 'This atom is on the left and this atom is on the right. *Therefore:* This atom is on the right and this atom is on the left'.

For the purposes of illustration, let's assign an interpretation to the individual constants a, b and c: suppose that whether X=R, L or S, $I(a,X)=0$, $I(b,X)=1$, and $I(a,X)=2$.

Now consider $V(Pa \wedge RightP(a))$. $V(Pa)=1$ since $[a]=I(a,L)=0$ and $0 \in I(P,L)$, so the left-hand side of the conjunction is true. On our specified interpretation $I(RightP,R)=\{0,1,2\}$ and $I(a,R)=0$. Since $0 \in I(Right,R)$, the right-hand side of the conjunction is true as well. Applying the truth-clause for $\wedge$ we see that the entire conjunction is true, i.e. $V(Pa \wedge RightP(a))=1$

Next consider $V(RightP(a))$. On our specified interpretation $I(RightP,S)=\emptyset$, and $I(a,S)=0$. We note that $0 \notin \emptyset$ and use the truth-clause for atomic sentences to conclude that $RightP(a)$ is false, i.e. $V(RightP(a))=0$. The interpretation I, and more generally the model of which it is a part, is a counterexample to ($\wedge E$). So ($\wedge E$) fails in at least one case, and hence is not a logical law.

The very same interpretation also provides counterexamples to ($\wedge I$) and the commutativity of conjunction, but in order to give a counterexample to (ID) we need to adapt our interpretations slightly, and allow not just position in a supersentence to determine truth-value, but also position in the argument. Suppose, for example, that we used the sentence positions P and C (for 'premises' and 'conclusion') as position arguments for I, and that our langauge contained predicates *PremP* and *ConcP*, whose extensions would vary with that position in the argument. This would allow us to define an interpretation, and a fortiori a model, on which $PremP(a) \vDash PremP(a)$ had a true premise but false conclusion, and give us a counterexample to (ID).

4 Sentences containing only logical expressions

The above models provide counterexamples to familiar logical principles by expanding the available interpretations for the non-logical expressions in a language. If our language is the simple one specified above, every sentence in it contains non-logical predicates and so is amenable to truth-value change by this method. Still, there are extensions of the language which contain sentences that lack a non-logical predicate or constant. Suppose, first, that we add a logical identity predicate, $=$. We make no change to the structure of the models, but extend the valuation function as follows:

$$V(t_1=t_2)=1 \text{ iff } [t_1]=[t_2]$$

The new symbol permits us to form the sentence $\forall x(x=x)$. This, one might think, will be a logical truth as usual. It doesn't *contain* any non-logical constants and so messing with the interpretation function will not affect its truth-value. Moreover, second, we might add the 0-place truth-functor $\top$, and the valuation clause:

$$V(\top) = 1$$

$\top$ is a sentence all on its own, and the valuation function ensures that it is true in all models, and so a logical truth. Perhaps there are logical truths after all, if our language is rich enough? Moreover, T allows us to say more: there are valid arguments with premises on the left of the turnstile;

$$\top \wedge \top \models \top$$

My responses to these two objections to strict nihilism will be different. To the first, I follow (Williamson, 2013) in holding that our variable assignments should assign the same kinds of objects to individual variables (and predicate-variables, if we are working in a second order logic) as our interpretation function assigns to individual constants (predicates). For this reason, when we add quantifiers and the corresponding variable assignments used in their interpretation, variable assignments too should take position in the sentence as an argument, in addition to the variable itself. Of course this makes space for the variable on the left of the identity sign to be assigned a different element from the variable on the right, and hence for the assignment to make $x = x$ false. $\forall x(x = x)$ would then fail too.

But this response does not cover $\top$ or $\top \wedge \top \models \top$. Here I think the objector is correct: this method of generating counterexamples to the usual logical principles does not cover $\top$, and so the argument has not taken us all the way to nihilism. I hold, however, that this is little comfort. The reason nihilism is troubling, if it is, is that it threatens the usefulness of logic. Nihilists will not be using their logic to do exciting metatheory, specify deductive theories, develop ambitious logicist projects, or provide a systematic approach to modal metaphysics. But if this is the problem with nihilism then it is similarly a problem for a related view that I will call logical *minimalism*: logical minimalism is the view that there are *hardly any* laws of logic. The view that the only laws of logic are constructed from truth-functional logical connectives *alone* (with no variables or non-logical expressions at all) is certainly such a view. Though it is not total nihilism, it might as well be.

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